References

[1]Hooper, W. E., "The Vibratory Airloading of Helicopter Rotors," *Vertica*, Vol. 8, No. 2, 1984, pp. 73-92.

[2]Shaw, J., "A Feasibility Study of Helicopter Vibration Reduction by Self-Optimizing Higher Harmonic Blade Pitch Control," M.S. Thesis, Dept. of Aeronautics and Astronautics, Massachusetts Inst. of Technology, Cambridge, MA, 1967.

[3]McHugh, F. J., and Shaw, J., "Helicopter Vibration Reduction with Higher Harmonic Blade Pitch," *Journal of the American Helicopter Society*, Vol. 23, No. 4, 1978, pp. 26-35.

[4]Shaw, J., and Albion, N., "Active Control of Rotor Blade Pitch for Vibration Reduction: A Wind Tunnel Demonstration," *Vertica*, Vol. 4, No. 1, 1980, pp. 3-11.

[5]Molusis, J. A., Hammond, C. E., and Cline, J. H., "A Unified Approach to the Optimal Design of Adaptive and Gain Scheduled Controllers to Achieve Minimum Helicopter Rotor Vibration," *Journal of the American Helicopter Society*, Vol. 28, No. 2, 1983, pp. 9-18.

[6]Molusis, J. A., "The Importance of Nonlinearity in the Higher Harmonic Control of Helicopter Vibration," American Helicopter Society, Alexandria, VA, 1983, pp. 624-647.

[7]O'Leary, J., and Miao, W., "Design of Higher Harmonic Control for the ABC," *Journal of the American Helicopter Society*, Vol. 27, No. 1, 1982, pp. 52-57.

[8]Johnson, W., "Self-Tuning Regulators for Multicyclic Control of Helicopter Vibration," NASA TP-1996, March 1982.

[9]Davis, M. W., "Refinement and Evaluation of Helicopter Real-Time Self-Adaptive Active Vibration Controller Algorithms," NASA CR-3821, Aug. 1984.

[10]Gupta, N. K., and Du Val, R. W., "A New Approach for Active Control of Rotorcraft Vibration," *Journal of Guidance and Control*, Vol. 5, No. 2, 1982, pp. 143-150.

[11]Du Val, R. W., Gregory, C. Z., Jr., and Gupta, N. K., "Design and Evaluation of a State-Feedback Vibration Controller," *Journal of the American Helicopter Society*, Vol. 29, No. 3, 1984, pp. 30-37.

[12]McKillip, R. M., Jr., "Periodic Control of the Individual-Blade-Control Helicopter Rotor," *Vertica*, Vol. 9, No. 2, 1985, pp. 199-225.

[13]Shaw, J., and Albion, N., "Active Control of the Helicopter Rotor for Vibration Reduction," *Journal of the American Helicopter Society*, Vol. 26, No. 3, 1981, pp. 32-39.

[14]Hammond, C. E., "Wind Tunnel Results Showing Rotor Vibratory Loads Reduction Using Higher Harmonic Blade Pitch," *Journal of the American Helicopter Society*, Vol. 28, No. 1, 1983, pp. 10-15.

[15]Lehmann, G., "A Digital System for Higher Harmonic Control of a Model Rotor," *Vertica*, Vol. 8, No. 2, 1984, pp. 165-181.

[16]Miao, W., Kottapalli, S. B. R., and Frye, H. M., "Flight Demonstration of Higher Harmonic Control (HHC) on S-76," *Proceedings of the 42nd Annual Forum of the American Helicopter Society*, American Helicopter Society, Alexandria, VA, 1986, pp. 777-791.

[17]O'Leary, J., Kottapalli, S. B. R., and Davis, M., "Adaptation of a Modern Medium Helicopter (Sikorsky S-76) to Higher Harmonic Control," *Proceedings of the 2nd Decennial Specialists Meeting on Rotorcraft Dynamics*, American Helicopter Society, Alexandria, VA, Nov. 1984, Paper No. 23.

[18]Walsh, D. M., "Flight Tests of an Open Loop Higher Harmonic Control System on an S-76A Helicopter," *Proceedings of the 42nd Annual Forum of the American Helicopter Society*, American Helicopter Society, Alexandria, VA, 1986, pp. 831-843.

[19]Wood, E. R., Powers, R. W., Cline, J. H., and Hammond, C. E., "On Developing and Flight Testing a Higher Harmonic Control System," *Journal of the American Helicopter Society*, Vol. 30, No. 1, 1985, pp. 3-20.

[20]Polychroniadis, M., and Achache, M., "Higher Harmonic Control: Flight Tests of an Experimental System on SA 349 Research Gazelle," *Proceedings of the 42nd Annual Forum of the American Helicopter Society*, American Helicopter Society, Alexandria, VA, 1986, pp. 811-820.

[21]Shaw, J., Albion, N., Hanker, E. J., Jr., and Teal, R. S., "Higher Harmonic Control: Wind Tunnel Demonstration of Fully Effective Vibratory Hub Force Suppression," *Journal of the American Helicopter Society*, Vol. 34, No. 1, 1989, pp. 14-25.

[22]Shaw, J., "Higher Harmonic Blade Pitch Control: A System for Helicopter Vibration Reduction," Ph.D. Thesis, Dept. of Aeronautics and Astronautics, Massachusetts Inst. of Technology, Cambridge, MA, 1980.

[23]McCloud, J. L., III, "The Promise of Multicyclic Control," *Vertica*, Vol. 4, No. 1, 1980, pp. 29-41.

[24]McCloud, J. L., III, and Weisbrich, A. L., "Wind-Tunnel Results of a Full-Scale Multicyclic Controllable Twist Rotor," *Proceedings of the 34th Annual Forum of the American Helicopter Society*, American Helicopter Society, Alexandria, VA, 1978 (Paper No. 78-60).

[25]McCloud, J. L., III, "An Analytical Study of a Multicyclic Controllable Twist Rotor," *Proceedings of the 31st Annual Forum of the American Helicopter Society*, American Helicopter Society, Alexandria, VA, 1975 (Paper No. 932).

[26]Hall, S. R., and Wereley, N. M., "Linear Control Issues in the Higher Harmonic Control of Helicopter Vibrations," *Proceedings of the 45th Annual Forum of the American Helicopter Society*, American Helicopter Society, Alexandria, VA, 1989, pp. 955-972.

[27]Gupta, N. K., "Frequency-Shaped Cost Functionals: Extension of Linear-Quadratic-Guassian Design Method," *Journal of Guidance and Control*, Vol. 3, No. 6, 1980, pp. 529-535.

[28]Kwakernaak, H., and Sivan, R., *Linear Optimal Control Systems*, Wiley, New York, 1972, pp. 281-296.

[29]Sarigul-Klijn, M. M., Kolar, R., Wood, E. R., and Straub, F. K., "On Chaos Methods Applied to Higher Harmonic Control," *Proceedings of the 46th Annual Forum of the American Helicopter Society*, American Helicopter Society, Alexandria, VA, 1990, pp. 79-98.

Autonomous Position and Velocity Determination in Interplanetary Space

John D. Vedder*

McDonnell Douglas Space Systems Company, Houston, Texas 77062

Introduction

THE purpose of this Note is to describe briefly a new method for autonomously estimating the position and velocity state vector of an interplanetary spacecraft. This method is based solely on a small number of sequential, time-tagged sightings of known objects with onboard optical equipment. These objects consist of the planets or their satellites and the brighter asteroids. The only qualifications for these objects are that they must be observable with onboard instrumentation, such as a star tracker; their ephemerides must be known onboard; and they must not be pairwise coplanar with the spacecraft's trajectory during the observation interval.

It is known that *simultaneous* sightings of two such objects against their stellar backgrounds can be used to determine the observer's position in interplanetary space, and detailed analyses of this process are available.[1,2] However, this process has some important limitations, including the following: sightings should ideally be simultaneous, necessitating multiple instruments and observers; no *direct* information about velocity is obtained; and any error in either of the two sightings will have a large effect on the result. Geometrically, this method fixes the observer's position as the point determined by the intersection of two straight lines in space.

The new method proposed here requires four *sequential*, time-tagged sightings of such objects to determine four lines in space. The direction of each line is extracted from the object's location against its observed stellar background. Knowing both this direction vector and the object's location (from its known ephemeris) defines the straight line in space. The observer's trajectory during the sighting interval is then approximated by the straight line that intersects each of the four known lines; actually two such solutions exist,[3] one represent-

Received June 30, 1992; presented as Paper 92-4599 at the AIAA Guidance, Navigation, and Control Conference, Hilton Head, SC, Aug. 10-12, 1992; revision received Sept. 1, 1992; accepted for publication Sept. 14, 1992.

*Specialist, Houston Division, 16055 Space Center Boulevard; currently Independent Consultant. Senior Member AIAA.

ing the actual trajectory and the other representing a spurious trajectory solution. Figure 1 illustrates the relevant geometry; however, the angular extent of the observer's trajectory during the sightings is greatly exaggerated for illustrative clarity. Knowing (in an ecliptic heliocentric coordinate system) both the equation of the straight line trajectory approximation and four time points on this line is sufficient to estimate the complete state vector. The direction of the straight line defines the *direction* of the velocity vector. The four time points on the line define estimates of the position vector at these times and thereby also define three estimates of the *magnitude* of the velocity vector. These multiple estimates can also be judiciously combined in various ways to produce a more accurate state vector estimate at a single time within the overall observation interval.

Analysis

Turning now to an analytical discussion of the problem of determining the equation of the straight line trajectory approximation, we note that any straight line in space can be represented as the intersection of two planes passing through the line. Hence each of the four lines ($k = 1, 2, 3, 4$) representing the four sightings can be represented by a pair of plane equations, such as

$$x + ya_{k1} + a_{k2} = 0$$
$$ya_{k3} + z + a_{k4} = 0 \tag{1}$$

where all of the coefficients are known and where, for convenience and without loss of generality, the planes are chosen so that as few coefficients as possible are nonvanishing. Clearly, Eq. (1) can also be interpreted as the projections of a line in space into the xy and yz planes; alternatively, the projections in the xy and xz planes or those in the yz and xz planes could be used if desired.

Each of four (i.e., $k = 1, 2, 3, 4$) successive pairs of plane equations (1) and (2) represents a single line passing through both the observer and the object sighted. Each of these four lines must intersect the assumed trajectory line

$$x + yc_1 + c_2 = 0$$
$$yc_3 + z + c_4 = 0 \tag{2}$$

where in this case the $\{c_i\}$ represents the four unknown coefficients of the trajectory line.

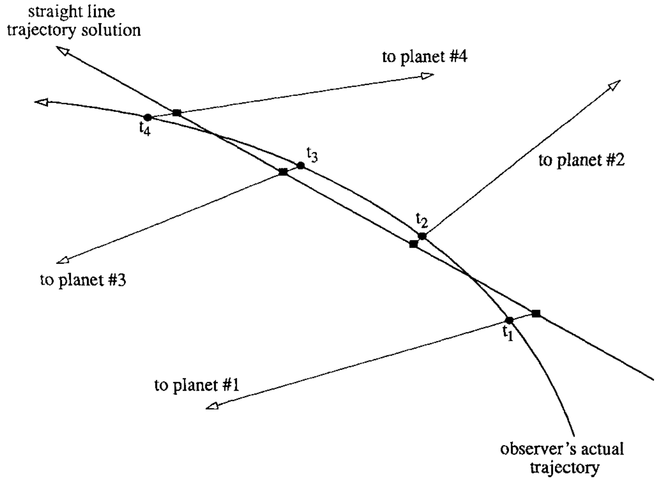

Fig. 1 Conceptual geometry of observations, with actual and computed trajectories.

The condition that each line [Eq. (1)] must intersect the line [Eq. (2)] in a point imples that the determinant of their coefficients must vanish in each case, i.e.,

$$\begin{vmatrix} 1 & a_{k1} & 0 & a_{k2} \\ 0 & a_{k3} & 1 & a_{k4} \\ 1 & c_1 & 0 & c_2 \\ 0 & c_3 & 1 & c_4 \end{vmatrix} = 0 \qquad k = 1, 2, 3, 4 \tag{3}$$

Evaluating Eq. (3) yields

$$a_{k1}(c_4 - a_{k4}) + a_{k2}(a_{k3} - c_3) - a_{k3}c_2 + a_{k4}c_1$$
$$= c_1c_4 - c_2c_3 \qquad k = 1, 2, 3, 4 \tag{4}$$

These four equations must be solved for the four unknown c_i to obtain the equation of the straight line representing the trajectory.

Rearranging the terms of Eq. (4) results in the matrix equation

$$\begin{bmatrix} a_{14} & -a_{13} & -a_{12} & a_{11} \\ a_{24} & -a_{23} & -a_{22} & a_{21} \\ a_{34} & -a_{33} & -a_{32} & a_{31} \\ a_{44} & -a_{43} & -a_{42} & a_{41} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} -a_{11}a_{14} + a_{12}a_{13} \\ -a_{21}a_{24} + a_{22}a_{23} \\ -a_{31}a_{34} + a_{32}a_{33} \\ -a_{41}a_{44} + a_{42}a_{43} \end{bmatrix}$$
$$= \begin{bmatrix} c_1c_4 - c_2c_3 \\ c_1c_4 - c_2c_3 \\ c_1c_4 - c_2c_3 \\ c_1c_4 - c_2c_3 \end{bmatrix} \tag{5}$$

which may be rewritten in symbolic matrix form as

$$\boldsymbol{Ac} + \boldsymbol{b} = \boldsymbol{g} \tag{6}$$

where $\boldsymbol{A}$ is the (4×4) matrix of coefficients in Eq. (5), $\boldsymbol{c}$ the (4×1) matrix of c in Eq. (5), and $\boldsymbol{b}$ the (4×1) matrix of bilinear coefficients on the left side of Eq. (5). In addition, $\boldsymbol{g}$ is the (4×1) matrix in Eq. (5) whose individual scalar elements g are identical, each equaling $(c_1c_4 - c_2c_3)$.

Solving Eq. (6) for $\boldsymbol{c}$, we obtain

$$\boldsymbol{c} = \boldsymbol{A}^{-1}\boldsymbol{g} - \boldsymbol{A}^{-1}\boldsymbol{b} = \boldsymbol{A}^{-1}(\boldsymbol{g} - \boldsymbol{b}) \tag{7}$$

although the $\{c_i\}$ are implicit in $\boldsymbol{g}$ as well as explicit on the left-hand side of Eq. (7). From Eq. (7), however, it is possible to obtain a single quadratic equation whose two solutions can be used to obtain the two possible solutions for the $\{c_i\}$. This quadratic equation is obtained from the definition of the scalar elements of $\boldsymbol{g}$ given previously, i.e.,

$$g = c_1c_4 - c_2c_3 \tag{8}$$

by substituting the c_i obtained from Eq. (7) into the right-hand side of Eq. (8). It is then possible to solve for the two possible values of g, since all other quantities in the equation are known. Knowing the two g, the two possible sets of $\{c_i\}$ are obtained from Eq. (7). And given the two $\{c_i\}$, the straight line trajectories are defined by Eq. (2). Thus, the equations for both the true and the spurious straight line trajectories have been explicitly obtained.

The solution just obtained is based on four independent, sequential sightings, and this is the minimum number required. However, if more than four sightings can be made, the solution given previously can straightforwardly be generalized to incorporate any number of additional sightings. These matrices become larger, and the system of equations becomes overdetermined. But optimal solutions can be obtained by replacing the ordinary inverse matrix in Eq. (7) with the Moore-Penrose pseudo-inverse.

Discussion

Although the solution just described is valid in the error-free case, this does not assure the method's usefulness in the real world. Obviously, the method's sensitivity to observational and other errors must be closely examined before drawing any conclusions about its usefulness. Another possible limitation stems from the fact that the major objects of the solar system are almost all near the ecliptic plane: if both the trajectory and the observed objects were all a single plane, the method would be invalid since no trajectory would be determined. Thus, being in proximity to a geometric degeneracy may increase the error sensitivity, although this would be mitigated if sightings of some objects, such as high-inclination asteroids, were used. In addition, the accuracy of the solution as a function of the angular distribution (with respect to the observer) of the four observed objects should be examined: better results should be obtained when their directions are spread out rather than concentrated.

Distinguishing between the true and spurious trajectory solutions is usually easy to do. However, in some cases the two solutions can be close to each other, and having a reliable means of distinguishing them in all cases is therefore highly desirable.

If, as appears likely, it is possible to cope with a considerable degree of actual trajectory curvature without excessively degrading accuracy, then the application of the method to autonomous navigation while in orbit around a planet may be feasible. At any rate, the method's accuracy as a function of trajectory curvature during the overall observation interval should be investigated.

Summary

A new method for autonomously determining an observer's state vector in deep space by optical means has been described. Although the method works in the error-free case, its usefulness in the presence of errors has not yet been established. Further investigation of the method is warranted to characterize its potential more fully.

References

[1]Vertregt, M., "Orientation in Space," *Journal of the British Interplanetary Society,* Vol. 15, pp. 324–338.

[2]Roy, A. E., *Orbital Motion,* 2nd ed., Wiley, New York, 1978, pp. 388–391.

[3]Sommerville, D. M. Y., *An Introduction to the Geometry of N Dimensions,* Dover, New York, 1958 (reprinted from the 1929 ed.), p. 11.